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Multipliers of multiple Fourier-Haar series

ABSTRACT
dissertation for the degree of Doctor of Philosophy (PhD) on specialty 6D060100 — Mathematics

The relevance of the research topic. The study of multipliers of Fourier series is an important direction in harmonic analysis. The great interest in this direction is explained by the fact that Fourier series multipliers are used in various sections of mathematics and in applied problems, as well as by the presence of unsolved problems requiring in-depth research. With the development of the theory of approximations by wavelets, there was an interest in the study of Fourier-Haar series.

The dissertation work is devoted to the study of multipliers of Fourier-Haar series in Lorentz spaces and in anisotropic Lorentz spaces.

The research objectives:

1. Study of the class of multipliers of Fourier-Haar series $m(L_{p,r} \to L_{q,s})$ in a more general situation, covering the case when $r > s$, $0 < r, s \le \infty$.

2. Investigation of multipliers of double Fourier-Haar series for functions from anisotropic Lorentz spaces. Obtaining necessary and sufficient conditions for the sequence $\lambda = \left\{\lambda_{k_1 k_2}^{j_1 j_2}\right\}$ to belong to the class $m\left(L_{\bar{p},\bar{r}} \to L_{\bar{q},\bar{s}}\right)$.

General research methodology. The main research apparatus is interpolation methods for anisotropic spaces, methods of the net spaces, Nikolsky-type inequalities, embedding theorems for anisotropic spaces.

Basic provisions for defense and the results of the study. The following main results of the dissertation the research were submitted for defense:

1. Interpolation theorem for anisotropic net spaces $N_{\bar{p},\bar{q}}(M)$, where M is the set of all rectangles in $\mathbb{R}^2$, $0 < \bar{p} = (p_1, p_2) \le \infty$, $1 \le \bar{q} = (q_1, q_2) \le \infty$.

2. The criterion for the function $f(x_1; x_2)$ to belong to the net space $N_{\bar{p},\bar{q}}(M)$ and the Lebesgue space $L_{\bar{p}}[0,1]^2$ with mixed metric, where $1 < \bar{p} < \infty$, $0 < \bar{q} \le \infty$, $\bar{p} = (p_1, p_2)$, $\bar{q} = (q_1, q_2)$, M is the set of all rectangles in $\mathbb{R}^2$. Hardy-Littlewood type theorem for multiple Fourier-Haar series.

3. Necessary and sufficient conditions for the sequence $\lambda = \{\lambda_k^j\}$ to belong to the class of multipliers of Fourier-Haar series $m(L_{p,r} \to L_{q,s})$.

4. Nikolsky-type inequality for multiple Fourier-Haar series. In particular, $\left\|S_{2^{k_1}2^{k_2}}(f)\right\|_{L_{\bar{q}}} = o\left(2^{k_1\left(\frac{1}{p_1}-\frac{1}{q_1}\right)} 2^{k_2\left(\frac{1}{p_2}-\frac{1}{q_2}\right)}\right)$ for $f \in L_{\bar{p},\bar{\tau}}[0,1]^2$.

5. Necessary and sufficient conditions for the sequence $\lambda = \left\{\lambda_{k_1 k_2}^{j_1 j_2}\right\}$ to belong to the multiplier class of multiple Fourier-Haar series $m\left(L_{\bar{p},\bar{r}} \to L_{\bar{q},\bar{s}}\right)$.

Theoretical and practical value. The results of the work are theoretical in nature and can be used in harmonic analysis, theory of differential equations, approximation theory, theory of functional spaces.

Structure and scope of the dissertation. The dissertation work, with a volume of 83 pages, consists of an introduction, four sections divided into subsections, conclusions and a list of sources used. The list of sources used includes 61 titles.

The main content of the work.

In the first section, anisotropic net spaces $N_{\bar{p},\bar{q}}(M)$ are studied, where M is the set of all rectangles in $\mathbb{R}^2$, $0 < \bar{p} = (p_1, p_2) \leq \infty$, $1 \leq \bar{q} = (q_1, q_2) \leq \infty$ and their interpolation properties. It is shown that the scale of the spaces $N_{\bar{p},\bar{q}}(M)$ is closed with respect to the multidimensional Fernandez interpolation method.

The main results of the second chapter are the Hardy-Littlewood theorems for multiple Fourier-Haar series in Lebesgue spaces $L_{\bar{p}}[0,1]^2$ with a mixed metric and anisotropic net spaces$N_{\bar{p},\bar{q}}(M)$.

In the third chapter, necessary and sufficient conditions are obtained for the sequence $\left\{\lambda_k^j\right\}_{k=0,j=1}^{\infty,2^k}$ to belong to the class $m(L_{p,r} \to L_{q,s})$, in particular, covering the case when $r > s$. he result obtained generalizes and supplements the earlier studies by O.V. Lelond, E.M. Semenova, S.N. Uksusov for the case $r \leq s$.

In the fourth chapter, an inequality describing the behavior of partial sums of double Fourier-Haar series is obtained:

$$\left(\sum_{k_2=0}^{\infty}\left(\sum_{k_1=0}^{\infty}\left(2^{k_1\left(\frac{1}{q_1}-\frac{1}{p_1}\right)+k_2\left(\frac{1}{q_2}-\frac{1}{p_2}\right)}\left\|S_{2^{k_1}2^{k_2}}(f)\right\|_{L_{\bar{q}}}\right)^{\tau_1}\right)^{\frac{\tau_2}{\tau_1}}\right)^{\frac{1}{\tau_2}} \leq c\|f\|_{L_{\bar{p},\bar{\tau}}}$$

In particular, this inequality implies that

$$\left\|S_{2^{k_1}2^{k_2}}(f)\right\|_{L_{\bar{q}}} = o\left(2^{k_1\left(\frac{1}{p_1}-\frac{1}{q_1}\right)}2^{k_2\left(\frac{1}{p_2}-\frac{1}{q_2}\right)}\right)$$

for $f \in L_{\bar{p},\bar{\tau}}[0,1]^2$.

The main result of the fourth chapter is to obtain necessary and sufficient conditions for the sequence$\lambda = \left\{\lambda_{k_1k_2}^{j_1j_2}\right\}$ to belong to the class $m\left(L_{\bar{p},\bar{r}} \to L_{\bar{q},\bar{s}}\right)$. In particular, the case is described when $\bar{s} < \bar{r}$, which is a new result in the one-dimensional case as well.

Approbation of the work. The main results of the dissertation work are presented and discussed

- at international scientific conferences: XV International Scientific Conference of Students, Undergraduates and Young Scientists "LOMONOSOV - 2019" (Nur-Sultan, 2019); «ACTUAL PROBLEMS OF ANALYSIS, DIFFERENTIAL EQUATIONS AND ALGEBRA» (EMJ-2019) dedicated to the 10th anniversary of the Eurasian Mathematical Journal (Nur-Sultan, 2019); XVI International Scientific Conference of Students, Undergraduates and Young Scientists "LOMONOSOV - 2020" (Nur-Sultan, 2020); International Scientific and

Practical Conference "Problems of Modern Fundamental and Applied Mathematics" (Nur-Sultan, 2021); Eurasian Youth Forum "Eurasia - a space of cooperation, peace and Harmony" dedicated to the 20th anniversary of the Kazakhstan branch of Lomonosov Moscow State University (Nur-Sultan, 2021); International Conference "Differential Equations and Related Topics" dedicated to the Ivan G. Petrovskii (Moscow, Russia, 2021);

- at the scientific regional seminar "Functional analysis and its applications" (heads: academician M. Otelbaev, academician R. Oinarov, professor E.D. Nursultanov, professor K.N. Ospanov, 2020);

- at the scientific seminar "Modern Problems of Mathematics" under the guidance of Professor E.D. Nursultanov, Kazakhstan branch of Moscow State University named after M.V. Lomonosov (2018, 2019, 2020).

- at the scientific seminar "Theory of trigonometric and orthogonal series" under the guidance of professors of the Department of Theory of Functions and Functional Analysis of Moscow State University named after M.V. Lomonosov M.K. Potapova, V.A. Skvortsova, T.P. Lukashenko, M.I. Dyachenko, Moscow, Russia (2020).

- at the city scientific seminar "Differential operators and their applications" / leaders of the seminar: Academician of the National Academy of Sciences of the Republic of Kazakhstan M. Otelbaev, Academician of the National Academy of Sciences of the Republic of Kazakhstan T.Sh. Kalmenov, Professor B.E. Kanguzhin, corresponding member NAS RK M.A. Sadybekov, Al-Farabi Kazakh National University, Institute of Mathematics and Mathematical Modeling (Almaty, 2022)

Publications. The results of the dissertation were published in 14 papers, including 5 papers in scientific rating journals, 9 papers in the collections of nine international scientific conferences.

Communication of this work with other research works. The topic of the dissertation research corresponds to the priority direction of development "Scientific research in the field of natural sciences", the specialized scientific direction "Fundamental and applied research in mathematics and mechanics". Some of the results of the dissertation were included in the interim report for 2021 on the project AP09260223 "Fourier transforms and multipliers of Fourier transforms of functions of many variables from anisotropic spaces" and report for 2020 on the project AP08053326 « Function space methods and their applications in harmonic analysis».